Moving Like a Solid Block

Dirk Helbing* and Bernardo A. Huberman+

* *II. Institute of Theoretical Physics, University of Stuttgart, Pfaffenwaldring 57/III, 70550 Stuttgart, Germany*

+ *Xerox PARC, 3333 Coyote Hill Road, Palo Alto, CA 94304, USA*

helbing@theo2.physik.uni-stuttgart.de, huberman@parc.xerox.com

Highway traffic is both a social phenomenon and a source of data for understanding the dynamics of systems made up of many individuals following their own behavioral rules. We present computer simulation results that point to the existence of a new and general phenomenon in mixed highway traffic under moderate density conditions. As the number of diverse vehicles in the road increases, there is a sharp transition into a highly coherent state of motion characterized by all vehicles having the same average speed and a very small variance in its distribution. This state, which is analogous to the motion of a solid block, is associated with a high and stable flow, and dissapears as the vehicle density increases beyond a critical value. The effect is observed in recent measurements of highway traffic of cars and trucks in the Netherlands and should be visible in other traffic situations as well.

Recent advances in multiagent simulation techniques [1–3] have made possible the study of realistic traffic patterns which can be used to test analytical theories based on driver's behaviors [3–6]. Such simulations also display a number of empirically known features of traffic flows [7], and are used to design traffic controls [8] that maximize the throughput of vehicles in heavily transited highways.

Vehicular traffic on highways is of interest because, in addition to its intrinsic economic value, it throws light on a number of dynamical social phenomena which result when individual drivers try to maximize their own utilities while dealing with those of others and the overall constraints imposed by traffic rules and physical limitations [9,10]. While individuals make their own decisions so as to when to pass, accelerate or even enter into a heavily transited multi-lane road [11–13], the externalities that they create make for traffic patterns that can at times be very regular, as exemplified by moving jam fronts [14] or synchronization patterns among adjacent lanes with low average velocity but high traffic flow [15].

In what follows we exhibit a new type of collective behavior that we discovered when studying the dynamics of a diverse set of vehicles, such as trucks and cars, with different velocities travelling through a highway. As the density of vehicles in the road increases, there is a transition into a highly coherent state characterized by all vehicles having the same average velocity and a very small dispersion around its value. In this regime, the whole set of vehicles behaves very much like a solid block, while the overall passing rate goes through a noticeable minimum.

Besides its intrinsic interest, this new phenomenon has practical implications as well, for it has been established that there is a correlation between the velocity variance of vehicles and the rate at which accidents take place [17]. The transition to a solid like flow becomes apparent when looking at the travel time distribution of all cars, comprising the overall dynamics on a freeway stretch. The distributions shown in Figure 1 were obtained by running computer simulations using a discrete follow-the-leader algorithm [18,19], which can be reformulated in terms of a cellular automaton [20]. Despite its simplifications, this model describes the empirical known features of traffic flows very well [20].

The algorithm distinguishes two neighboring lanes $i \in \{0, 1\}$ of an unidirectional freeway. Both are subdivided into sites $z \in \{1, 2, \dots, L\}$ of equal length $\Delta x = 2.5\,\mathrm{m}$. Each site is either empty or occupied by a vehicle of type $a \in \{\text{car, truck}\}$ with velocity $v = u\Delta x/\Delta t$, so that it moves $u \in \{0, 1, \dots, u_a^{\max}\}$ sites per update step Δt. Cars and trucks are characterized by different optimal (i.e. maximally safe) velocities $U_a(d_+)$ with which the vehicles would like to drive at a distance d_+ to the vehicle in front. The positions $z(T)$, velocities $u(T)$, and lanes $i(T)$ of all vehicles are updated every time step $\Delta t = 1\,\mathrm{s}$ at times $T \in \{1, 2, \dots\}$ [16] according to the following successive steps. We will denote the position, velocity, and distance of the respective leading vehicle $(+)$ or following vehicle $(-)$ on lane $i(T)$ by $z_\pm$, $u_\pm$, and $d_\pm = (z_\pm - z)$, on the adjacent lane $1 - i(T)$ by $z'_\pm$, $u'_\pm$, and $d'_\pm = (z'_\pm - z)$.

1. *Determine the potential velocities* $u(T+1)$ and $u'(T+1)$ on the present and the adjacent lane according to the *acceleration law*

$$u^{(\prime)}(T+1) = \left\lfloor \lambda U_a(d_+^{(\prime)}(T)) + (1-\lambda)u(T) \right\rfloor,$$

where the floor function $\lfloor x \rfloor$ is defined by the largest integer $l \leq x$. This describes the typical follow-the-leader behavior of driver-vehicle units. Delayed by the reaction time Δt, they tend to move with their optimal velocity U_a, but the adaptation takes a certain time $\tau = \lambda \, \Delta t$ because of the vehicle's inertia.

2. *Change lane* (in accordance with American rules) if the following incentive and safety criteria are fulfilled: Check if the distance $d'_{-}(T)$ to the following vehicle on the neighboring lane is greater than the distance $u'_{-}(T+1)$ that this vehicle is expected to move within the reaction time Δt (safety criterion 1). If so, the difference $D = d'_{-}(T) - u'_{-}(T+1) > 0$ defines the backward surplus gap. Next, look if you could go faster on the adjacent lane (incentive criterion). Finally, make sure that the relative velocity $[u'_{-}(T+1) - u'(T+1)]$ would not be larger than $D + m$ (safety criterion 2), where the magnitude of the parameter $m \geq 0$ is a measure of how relentless drivers are in overtaking. A value $m > 0$ implies that drivers are ready to enforce a braking maneuver of the follower on the destination lane.

3. If, on the updated lane $i(T+1)$, the corresponding potential velocity $u(T+1)$ or $u'(T+1)$ is positive, diminish it by 1 with probability p in order to consider cases of *delayed adaptation* due to reduced attention of the driver.

4. Update the vehicle position according to the *equation of motion* $z(T+1) = z(T) + u(T+1)$.

The scenario that we considered is that of a circular one-way two-lane highway of 10 kilometer length in which 98% cars and 2% trucks move with different velocities. Our simulations demonstrate that, at small densities, the travel time distribution has two narrow peaks at the maximum velocities of cars and trucks, since cars can overtake trucks without prior slowdowns (Fig. 1a). With growing but still moderate density, the arrival times of trucks remain about the same because they tend to have large headways due to their slow speed, whereas the arrival times of cars grow. As the neighboring lane may be blocked, some cars will have to temporarily slow down to the trucks' speed. In this regime a car can get stuck behind a truck for a long time, as overtaking requires a particularly large gap due to the high relative velocity between vehicles in adjacent lanes. Therefore, the travel time distribution develops a second peak around the truck peak and a continuum of intermediate travel times in between (Fig. 1b). At a certain critical density the shifted car peak merges with the truck peak (Fig. 1c). In this state, trucks and vehicles move practically at the same speed with very small dispersion, almost like a solid block of particles. If the density is slightly increased, this highly correlated state of motion breaks down, and a broad distributions of travel times results (Fig. 1d).

The transition to the solid-like state of traffic flow at medium densities is also seen in Figure 2, which displays the velocity-density relations of cars and trucks by dashed lines. Whereas the velocity-density relation of cars decreases monotonically, the average velocity of trucks stays almost constant up to the critical density of 24 vehicles per kilometer and lane, and then it falls abruptly. This is a typical feature of a phase transition, where the slope changes discontinuously. Moreover, between 22 and 24 vehicles per kilometer and lane, cars and trucks move with almost identical velocities. Therefore, large gaps will rarely develop so that the lane-changing rate drops by about one order of magnitude, as shown in Figure 3. Without opportunities for overtaking however, the vehicles have to go at the speed of the trucks, which closes the feedback-loop that causes the transition. High lane-changing rates presuppose the frequent occurence of large gaps, and they can only appear at sufficiently small densities or for unstable stop-and-go traffic developing at densities above 25 vehicles per kilometer and lane.

As shown in Figure 4, our prediction of the phase transition into a highly coherent state is supported by empirical data obtained from highway traffic in the Netherlands. Not only does the average velocity $V_{\rm T}$ of trucks stay constant up to a density of about 25 vehicles per kilometer and lane, where it starts to decrease significantly (Fig. 4a), but also the difference $(V_{\rm C} - V_{\rm T})$ between the average velocities $V_{\rm C}$ and $V_{\rm T}$ of cars and trucks shows the predicted minimum. Although the minimum is broader and less steep than in our simulations, the decrease of the *relative* difference $(V_{\rm C} - V_{\rm T})/V_{\rm T}$ is about 50%, i.e. quite noticeable [21]. Simulations with higher values of the fluctuation parameter ($p \approx 0.2$) yield a more quantitative agreement with the data and suggest that we are confronted with a *noisy* transition. This interpretation is also supported by the relative fluctuation of vehicle speeds (Fig. 4b), which shows a minimum at a density of about 25 vehicles per kilometer per lane, while remaining finite.

In summary, we have presented a novel effect in highway traffic that consists in the formation of a coherent state out of a disorganized flow of vehicles. This transition takes place without any global coordinator and is the result of the interaction of many agents trying to maximize their own utilities (to get ahead as fast as possible, but safely). This effect has been observed in real highway traffic, and our interpretation suggests that it is largely independent of the chosen driver-vehicle model. At a practical level, since there is a correlation between a high velocity variance of vehicles and their rate of accidents [17], it is important to design highway controls that will lead to traffic moving like a solid block. Notice that this kind of traffic is also characterized by a high and stable flow. It would be interesting to see if the spontaneous appearance of a highly coherent state is also found in other social or biological systems, such as correlated motion in pedestrian crowds [22], cell colonies [23], or flocking phenomena [24].

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FIGURES

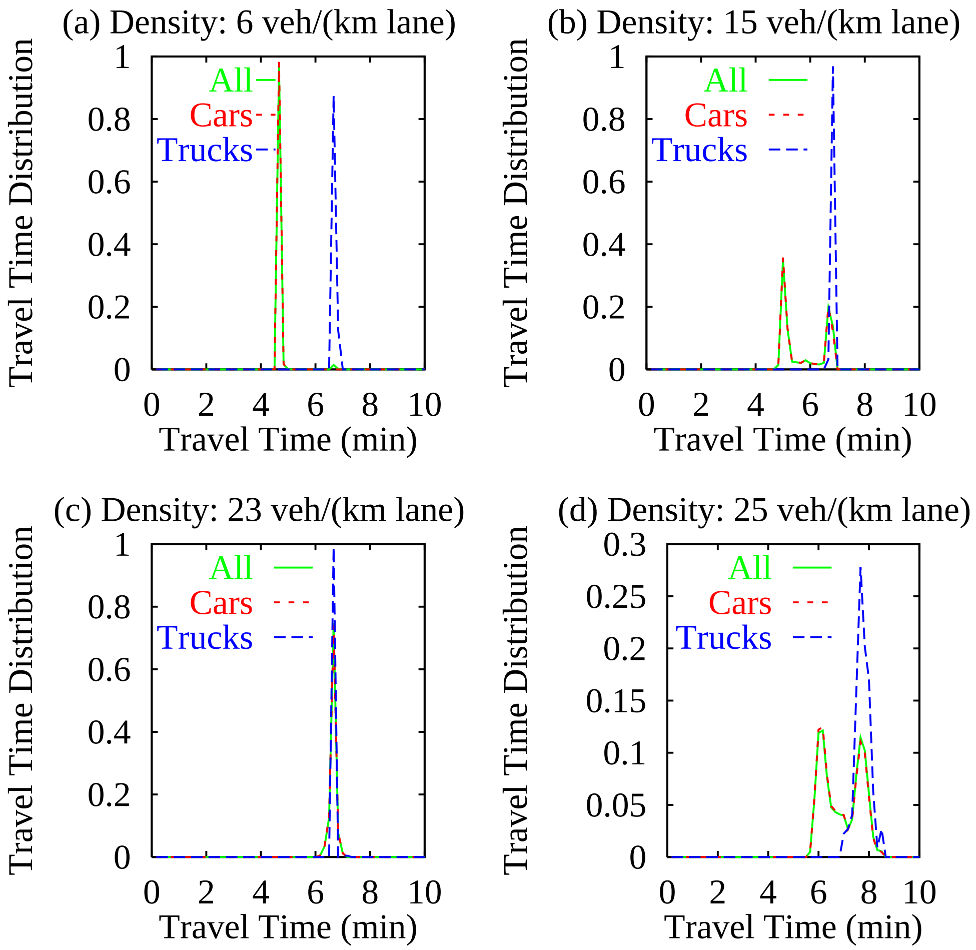

FIG. 1. Simulated travel time distributions of all vehicles, cars, and trucks at different vehicle densities for the model parameters $\lambda = 0.77$, $p = 0.05$, and $m = 2$.

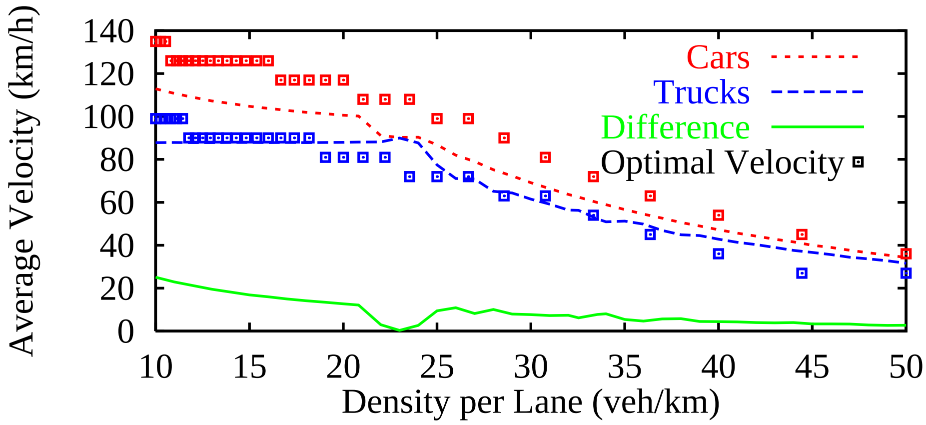

FIG. 2. The numerically determined velocity-density relations for cars and trucks (dashed lines) come particularly close to each other at densities between 22 and 24 vehicles per kilometer and lane (solid line). Obviously, this is not enforced by the particular choice of the so-called optimal velocity-density relations specified by the model (boxes). The parameter values were chosen as in Fig. 1.

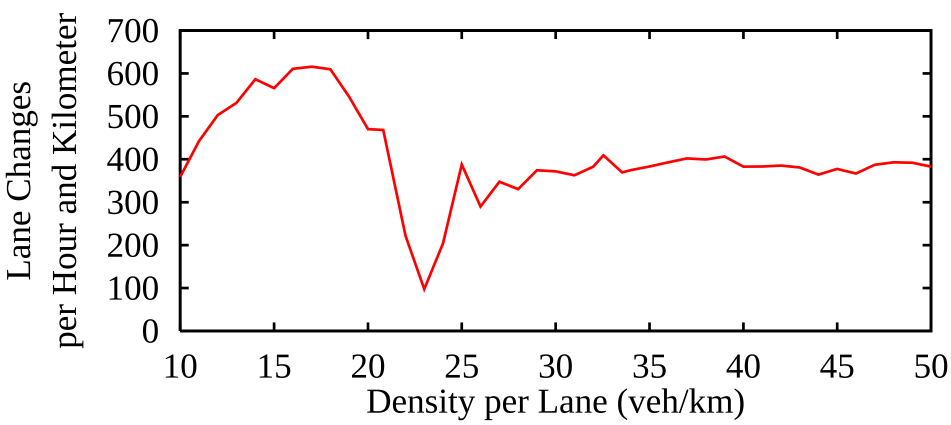

FIG. 3. The simulated lane-changing rate in dependence of vehicle density shows a distinct minimum at a the critical density of 23 vehicles per kilometer and lane (same model parameters as in the previous figures).

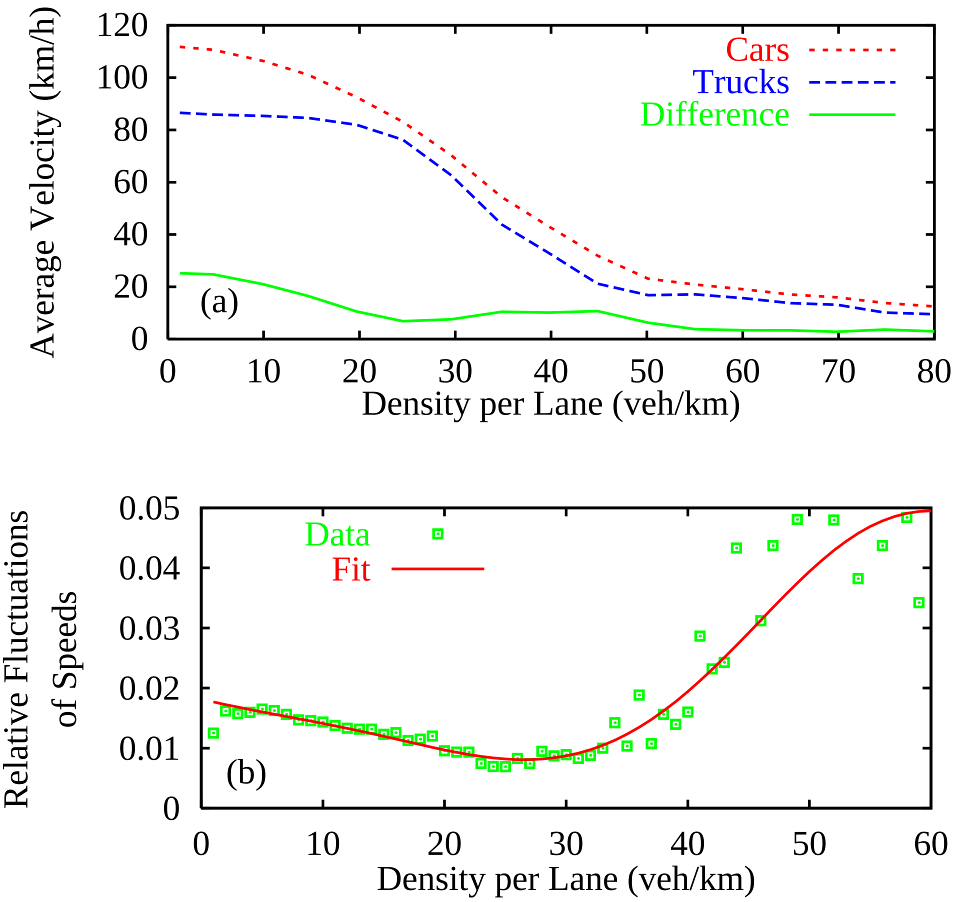

FIG. 4. (a) Empirical velocity-density relations for mixed traffic on the Dutch two-lane freeway A9. (b) Density-dependent relative fluctuation of vehicle speeds (defined as velocity variance divided by the square of average velocity). The data correspond to mean values of one-minute averages.