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Controls for unstable structures

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ABSTRACT

We study the behavior of several organizations for a market based distributed control of unstable physical systems and show how a hierarchical organization is a reasonable compromise between rapid local responses with simple communication and the use of global knowledge. We also introduce a new control organization, the multihierarchy, and show that it uses less power than a hierarchy in achieving stability. The multihierarchy also has a position invariant response that can control disturbances at the appropriate scale and location.

Keywords: distributed control organizations, smart matter, multiagent systems

1. INTRODUCTION

Embedding microscopic sensors, computers and actuators into materials allows physical systems to actively monitor and respond to their environments in precisely controlled ways. This is particularly so for microelectromechanical systems (MEMS)[1, 2, 6] where the devices are fabricated together in single silicon wafers. Applications include environmental monitors, drag reduction in fluid flow, compact data storage and improved material properties.

In many such applications the relevant mechanical processes are slow compared to sensor, computation and communication speeds. This gives rise to a *smart matter* regime, in which control programs can execute many steps within the time needed for responding to mechanical changes. A key difficulty in realizing smart matter's potential lies in the design of the control programs. This is because one needs to robustly coordinate a physically distributed, real-time response, to a system with many elements that can exhibit failures, delays and a limited ability to accurately model the system's behavior in an unpredictable environment. These constraints limit the effectiveness of conventional control algorithms, which rely on a single global processor with rapid access to the full state of the system and detailed knowledge of its behavior.

A robust approach to controlling such systems uses a collection of autonomous agents, each of which can deal with a limited aspect of the overall control problem. Individual agents can be associated with each sensor or actuator in the material. This leads to a community of computational agents which, in their interactions, strategies, and competition for resources, resemble natural ecosystems[14], and which can be used to control distributed systems[10].

Multiagent systems have been extensively studied in the context of distributed problem solving[4, 7, 16]. They have also been applied to problems involved in acting in the physical world, such as distributed traffic control[18],

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flexible manufacturing[23], the design of robotic systems[19, 26], and self-assembly of structures[21]. However, the use of multiagent systems for controlling smart matter is a challenging new application due to the very tight coupling between the computational agents and their embedding in physical space. Specifically, in addition to computational interactions between agents from the exchange of information, there are mechanical interactions whose strength decreases with the physical distance between them.

In this paper we study the effect that different control organizations have on achieving stable behavior of an intrinsically unstable dynamical system. This is a particularly challenging problem, for in the absence of controls, the physics of an unstable system will rapidly drive it away from the desired configuration. That is the case, for example, of a structural beam whose load is large enough to cause it to buckle and break. In such cases, weak control forces, if applied properly, can counter departures from the unstable configuration while they are still small. Successful control in this case not only leads to a virtual strengthening of the material but also to rapid changes of the system into other desired configurations.

The paper is organized as follows. We first introduce the unstable physical system that we attempt to control. We then describe a market system for trading power among actuators that can provide distributed control in the face of local failures and limited information about the state of the system. Finally we analyze the performance of several control organizations of this multiagent system and compare their power needs in order to achieve a desired level of control. We then show how a particularly novel structure, the multihierarchy, can achieve desirable control levels while using less power than other organizations.

2. SYSTEM DESCRIPTION

The system we studied consists of n mass points connected to their neighbors by springs. In addition, a destabilizing force proportional to the displacement acts on each mass point. This models the behavior of unstable fixed points: the force is zero exactly at the fixed point, but acts to amplify any small deviations away from the fixed point. This system can be construed as a linear approximation to the behavior of a variety of dynamical systems near an unstable fixed point. In the absence of control, any small initial displacement away from the fixed point grows rapidly.

The dynamical behavior of the chain is described by

1. the number of mass points: n
2. spring constants: k
3. a destabilizing force: f
4. a damping coefficient: g

and the elements have unit mass. We assume that sensors and actuators are embedded in this system at various locations. Associated with these devices are computational agents that use the sensor information to determine appropriate actuator forces. The overall system dynamics can be viewed as a combination of the behavior at the location of these agents and the behavior of the material between the agent locations. The dynamics for the latter consists of high frequency oscillations that are not important for the overall stability[11] of the system. This is because stability is primarily determined by the behavior of the lowest frequency modes. We assume that there are enough agents so that their typical spacing is much smaller than the wavelengths associated with these lowest modes. Hence, to focus on the lower frequency dynamics it is sufficient to characterize the system by the displacements at the locations of the agents only. In this case, the high-frequency dynamics of the physical substrate between agents does not significantly affect overall stability. Instead, the substrate serves only to couple the agents' displacements. The resulting dynamics of the unstable chain is given by:

$$\begin{aligned}\frac{dx_i}{dt} &= v_i \\ \frac{dv_i}{dt} &= k(x_{i-1}-x_i) + k(x_{i+1}-x_i) + fx_i - gv_i + H_i\end{aligned} \tag{1}$$

where x_i is the displacement of mass point i, v_i is the corresponding velocity, and $x_0 = x_{n+1} = 0$ is the boundary condition. The H_i term in Eq. (1) is the additional control force produced by the actuator attached to mass point i. We assume that the magnitude of this control force is proportional to the power P_i used by the actuator and we use a unit proportionality factor.

For this system, the control problem consists in determining how hard to push on the various mass points to maintain them at an unstable fixed point. Solving this problem can involve various goals, such as maintaining stability in spite of perturbations typically delivered by the system's environment, using only weak control forces so that the actuators are easy and cheap to fabricate, continuing to operate even with sensor noise and actuator failures, and being able to program the system without requiring a detailed physical model.

3. USING MARKETS FOR DISTRIBUTED CONTROL

While in principle an omniscient central controller with a perfect system model and unlimited computational capability could optimally control it, in practice such detailed knowledge of the system is seldom available. This is especially true in the case of mass production of smart materials, where manufacturing tolerances and occasional defects will cause the physical system to differ somewhat from its nominal specification. Instead, partial information about local changes in the variables is the only reliable source that can be used for controlling smart matter. In particular, price mechanisms perform well compared to other feasible alternatives[5, 15] for a variety of multiagent tasks. It is thus of interest to see how they perform in the new context of multiagent control of smart matter.

Computational markets can successfully coordinate asynchronous operations in the face of imperfect knowledge and changes in the environment[3, 13, 17, 22, 24, 25]. As in economics, the use of prices provides a flexible mechanism for allocating resources, with relatively low information requirements[9]: a single price summarizes the current demand for each resource.

In the market control treated here, actuators, or the corresponding mass points to which they are attached, are treated as consumers. The external power sources are the producers and as such are separate from consumers. All consumers start with a specified amount of money and all the profit that the producers get from selling power to consumers is equally redistributed to the consumers. This funding policy implies that the total amount of money in the system will stay constant.

In the spirit of a smart matter regime, where control computations are fast compared to the relevant mechanical time scales, we assume a market mechanism that rapidly finds the equilibrium point where overall supply and demand are equal. This equilibrium determines the price and the amount of power traded. Each actuator gets the amount of power that it offers to buy for the equilibrium price and uses this power to push the unstable chain.

Each consumer buys an amount of power, P_i, that depends on its associated utility function, U_i, which reflects a trade-off between using power to act against a displacement and the loss of wealth involved. While a variety of utility functions are possible, a particularly simple one for agent i, expressed in terms of the price of the power, p, and the agent's wealth, w_i, is:

$$U_i = -\frac{1}{2w_i}pP^2 + bP|X_i| \tag{2}$$

where

$$X_i = \sum_{j=1}^{n} a_{ij}x_j \tag{3}$$

is a linear combination of the displacements of all mass points that provides information about the chain's state and represents the underlying organizational structure of the control method. The parameter b determines the relative importance to an agent of responding to displacements compared to conserving its wealth for future use.

All the actuators push so as to reduce the value of X_i. We also use an ideal competitive market in which each consumer and producer acts as though its individual choice has no affect on the overall price, and agents do not account for the redistribution of profits via the funding policy. Thus the consumer's demand function is obtained by maximizing its utility function as a function of power, which yields

$$P_i(p) = b|X_i|\frac{w_i}{p} \tag{4}$$

This demand function causes the agent to demand more power when the displacement it tries to control is large. It also reflects the trade-off involved in maintaining wealth: demand decreases both with increasing price as well as when agents have little wealth. The overall demand function for the system is just the sum of these individual demands, giving

$$P^{\text{demand}}(p) = \frac{b}{p}\sum_{i=1}^{n}|X_i|w_i \tag{5}$$

As to the functional form of the supply function, each producer tries to maximize its profit ρ given by the difference between its revenue from selling power and its production cost $C(P)$: $\rho = pP - C(P)$. To provide a constraint on the system to minimize the power use, we select a cost function for which the cost per unit of power, $C(P)/P$, increases with the amount of power required. A simple example of such a cost function is given by

$$C(P) = \frac{1}{2a}P^2 \tag{6}$$

with the parameter a reflecting the relative importance of conserving power and maintaining stability. Similarly, we obtain the producer's supply function by maximizing its profit, which gives:

$$P(p) = ap \tag{7}$$

For simplicity, we assume that all producers have the same cost, so that the overall supply function is

$$P^{\text{supply}}(p) = nap \tag{8}$$

From this, we can obtain the price and amount of traded power as the point where the overall demand curve intersects the overall supply curve. Specifically, the price at which power is traded is given by $P^{\text{demand}}(p) = P^{\text{supply}}(p)$. For our choices of the utility and cost functions, this condition can be solved analytically to give

$$p_{trade} = \sqrt{\frac{b}{na}\sum_{i=1}^{n}|X_i|w_i} \tag{9}$$

Given this equilibrium price, agent i then gets an amount of power equal to $P_i(p_{trade})$ according to Eq. (4).

The final aspect of the market dynamics is how wealth changes with time. Using a funding policy such that all expenditures are returned equally to the agents in the system gives a dynamical equation of the form

$$\begin{aligned}\frac{dw_i}{dt} &= -pP_i(p) + \frac{1}{n} p P^{\text{demand}}(p) \\ &= -b|X_i|w_i + \frac{b}{n}\sum_{j=1}^{n} |X_j|w_j\end{aligned} \tag{10}$$

This model establishes a particular multiagent control method. Its performance will depend on the organizational structure through the choice of the quantity that the agents respond to i.e., the combinations of displacements of individual elements, X_i.

4. MARKET ORGANIZATIONS

We now determine the role that the underlying organizational structure of the market plays in the performance of the control system. Interpreting the a_{ij}'s in Eq. (3) as a description of how important it is for agent i (actuator) to get information from agent j (sensor) when deciding how much power to request, allows us to study different organizations by changing the structure of this interaction matrix. In what follows we will focus on 4 different organizational structures; (1) local, (2) global, (3) hierarchical and (4) multihierarchical[11]. These structures are represented by interaction matrices shown in Fig. 1. In the case of a local structure every agent uses only his own information, i.e., $a_{ij} = \delta_{ij}$ which is 1 when $i = j$ and zero otherwise. A global structure allows all agents to access all of the available sensor information. Thus the matrix elements, a_{ij}, for the global structure are calculated by fitting the unstable modes to the chain's state using the sensor data of all agents.

In the case of both a hierarchical and a multihierarchical structure, the control system is divided into a number of levels, with "managers" at each level responsible for communicating with subordinate agents below and superior agents above. In terms of information flow, the managers can aggregate the states of lower level agents and make this aggregate information available to agents in other parts of the system. In this way, each agent has, in addition to its own state, some averaged information about the state of the system on larger scales. Hierarchical organizations for control have been used in other contexts, such as controlling vibrations in a stable system[8, 12].

The multihierarchy consists of a structure of interleaved hierarchies having the advantage of solving the difficulty associated with the hierarchy of occasional mismatches between the physical and organizational distances between agents. In particular, agents near an organizational boundary of a high level part of the hierarchy require many levels in the hierarchy to communicate with some of their immediate physical neighbors. This introduces an inhomogeneity in the response of the hierarchical system in that some medium-scale perturbations can be controlled entirely within a single part of the hierarchy while others, crossing these high level boundaries, require additional levels of communication and aggregation of information.

We now compare the performance of these structures in terms of overall used power, average displacement and time to reduce the average displacement below a given value.

We studied[a] a chain composed of 27 mass points, all of them having unit mass and connected by springs with a spring constant of value 1 and damping coefficient 0.1. The destabilizing force coefficient is 0.05, which is sufficient to make the system unstable when there is no control force. Specifically, this makes only the lowest

[a] We used a standard ordinary-differential-equation solver for integrating the equations of motion[20].

mode of the system unstable. All agents start with an initial wealth of 50 money units. In the cost function of Eq. (6) we use $a = 0.05$. In order to compare the 4 structures in a fair way, we ran several simulations for each organization and searched for the b value in Eq. (2) that results in successful control with the least amount of power used. This results in using $b = 3.9 \times 10^{-5}$, $3.1 \times 10^{-5}, 5.0 \times 10^{-5}, 3.7 \times 10^{-5}$ for the local, global, hierarchical and multihierarchical structures, respectively. This allows us to compare the performance of the different structures in a fair way.

For definiteness, we chose an initial condition where a single element in the middle of the chain had a unit displacement and all other elements of the chain have no displacement. This initial condition includes a contribution from all modes of the system, specifically including the lowest mode, which is unstable when there is no control force. Since in this case there is a single unstable mode, the calculation of the a_{ij}'s for the global, hierarchical and multihierarchical structure are only based on a fit of the first unstable mode. Fig. 1 shows the interaction matrices corresponding to the different structures that we studied.

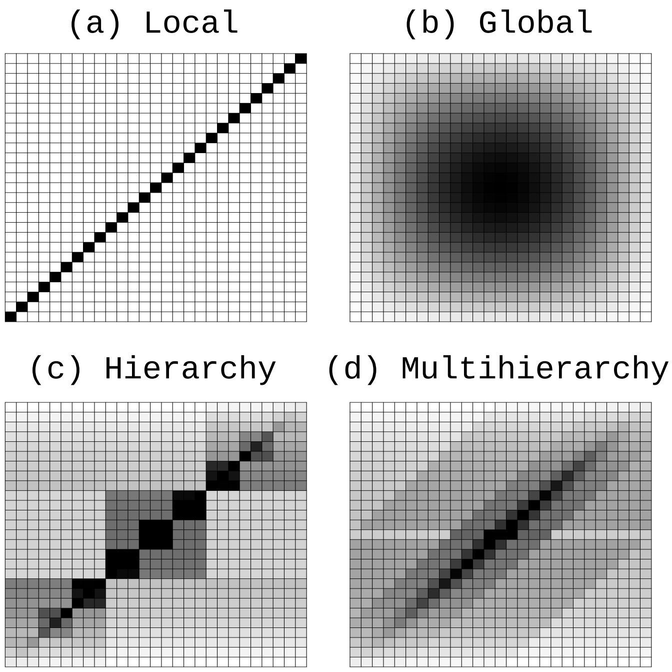

Fig. 1. Interaction matrices of four different organizations. For each organization's plot, starting in the lower-left corner, columns represent agent *i* and rows represents agent *j*, with *i* and *j* going from 1 to 27. The shade in each small square characterizes the interaction between agent *i* and agent *j* at that position. Black represents a strong connection, and gray a weaker one. (a) corresponds to a local structure where every agent listens only to itself and ignores all other agents in the system. The interaction with other agents is only mediated by the springs that connect elements. (b) depicts the global case, whereas (c) and (d) correspond to a hierarchical and a multihierarchical organization with 3 levels and a branching ratio of 3.

The performance of these organizations when confronted with reducing the initial value displacement is shown in Fig. 2. As can be seen, the time evolution of the average displacement differs slightly for different organizations.

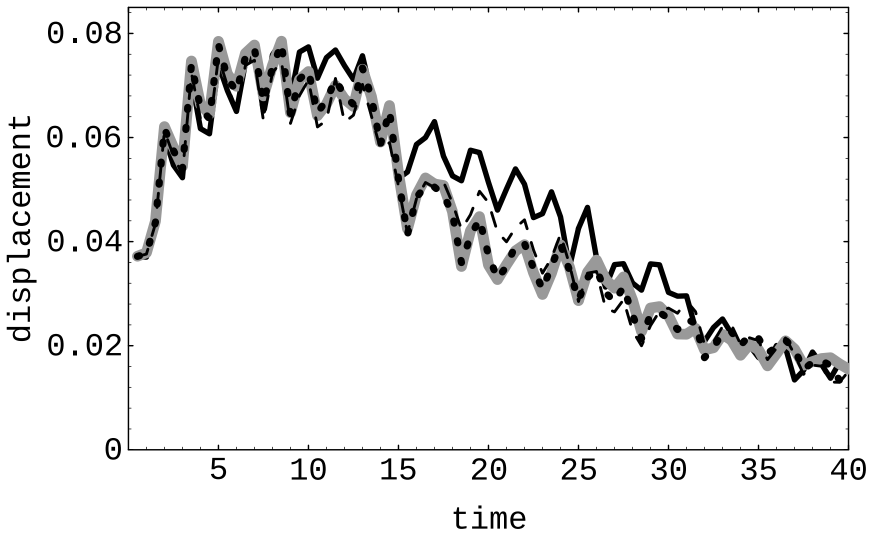

Fig. 2. Average displacements of the unstable chain when using different organizational control structures. The solid, black curve represents the local structure. The global structure is represented by the solid, gray curve and the dashed and the dotted curves present the hierarchy and the multihierarchy, respectively.

On the other hand, since power is the good traded in the market one expects that the different organizational structures will exhibit different behavior in their power usage in Fig. 3. As one could have assumed, the global control uses less power than the local control whereas the hierarchical and multihierarchical structure lie somewhere between in their power use. Notice that the global structure needs less power than the local one, except for a small time at the beginning, and on average the multihierachy uses less power than the hierarchy.

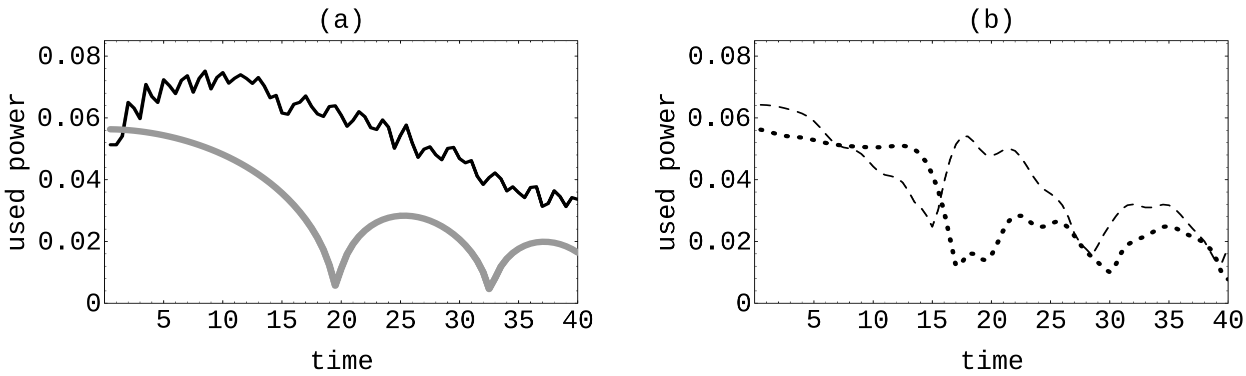

Fig. 3. Time evolution of the power used by different control organizations. In a) we compare the power usage of a local (solid, black) and a global (solid, gray) organization. In b) we compare the hierarchical (dashed) with the multihierarchical (dotted) organization.

Since the power usage curves have different functional forms, for purposes of comparison it is more convenient to plot the overall power used to push the average displacement of the chain below a given threshold value. This is shown in Fig. 4. The x-axis denotes the threshold value for the average displacement and the y-axis corresponds to the overall power needed to reduce the average displacement below that value. As can be seen, the global structure (solid, gray curve) performs best, although the multihierarchy (dotted curve) shows almost the same performance. The local organization (solid, black curve) uses the most power whereas the hierarchical structure (dashed) lies somewhere between.

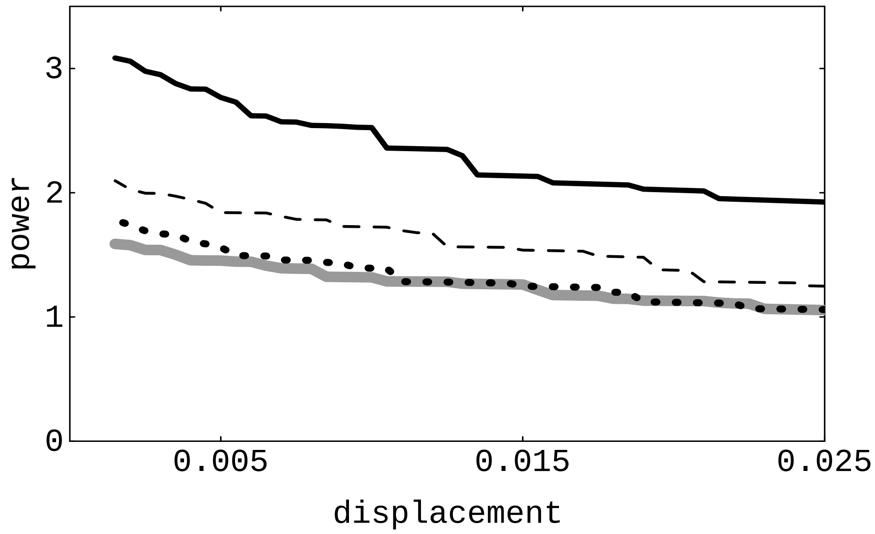

Fig. 4. Overall power usage for the different organizational structures to reduce the average displacement below a given threshold that is given by the value on the x-axis.

5. DISCUSSION

In this paper we presented a novel mechanism for controlling unstable dynamical systems that uses a multiagent approach combined with a market mechanism for trading power among actuators. This allows for distributed control of an unstable system in spite of limited information about its dynamical state. We also studied the effectiveness of different market organizations in achieving control, and we compared their relative performance. While we noticed that the average excursions of the controlled system do not depend significantly on the control organization, the power usage does. In particular a global control organization uses the least amount of power to control the unstable system while also being capable of reducing the average displacement of the chain. This stems from the fact that the agents have access to global information while using a model that describes the physical connections within the system. While the global approach is appropriate for small systems with a well defined dynamical model, it becomes increasingly difficult to apply it to larger systems where the available information about the system's state might be limited.

This led to consider alternative organizations, such as a local control, and more interconnected ones, such as hierarchies and multihierarchies. We then showed that the use of hierarchical and multihierarchical structures lead to a better performance than using a local structure, in terms of the amount of power needed for controlling the unstable system is needed. Moreover a multihierarchy control organization uses less power than the simple hierarchical structure. This is due to the position invariant response of a multihierarchy, which can therefore control disturbances at appropriate scales. This difference may become even more significant if each level of managers introduces some delay in communicating the aggregate information.

An extension of this research that we are now pursuing consists in the use of a learning mechanism that can improve the performance of the system by changing the market organization in time. This would allow the system to dynamically find those structures most suited for particular situations and to change them as necessary. Equally important, it could lead to the discovery of novel organizational structures that cannot be anticipated from a priori analysis of systems that by their very nature are imperfect.

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