Market Organizations for Controlling Smart Matter

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Abstract

We present a market solution to the problem of maintaining a spatially extended physical system near an unstable configuration. Computational agents representing individual controllers bid to buy or sell power that is used to push the system to a desired unstable configuration. Simulation results show that the auction leads to a stable control of an otherwise unstable system by focussing the power in those parts of the system where is most needed. We also studied the effectiveness of different market organizations in achieving control, and we noticed that the nearest neighbor structure uses the least amount of power to control the unstable system while also capable of reducing the average displacement of the chain.

1. Introduction

Embedding microscopic sensors, computers and actuators into materials allows physical systems to actively monitor and respond to their environments in precisely controlled ways. This is particularly so for microelectromechanical systems (MEMS) [1, 2, 6] where the devices are fabricated together in single silicon wafers. Applications include environmental monitors, drag reduction in fluid flow, compact data storage and improved material properties.

In many such applications the relevant mechanical processes are slow compared to sensor, computation and communication speeds. This gives a *smart matter* regime, where control programs execute many steps within the time available for responding to mechanical changes. A key difficulty in realizing smart matter's potential is developing the control programs. This is due to the need to robustly coordinate a physically distributed real-time response with many elements in the face of failures, delays, an unpredictable environment and a limited ability to accurately model the system's behavior. When the system contains many elements, these characteristics limit the effectiveness of conventional con-

trol algorithms, which rely on a single global processor with rapid access to the full state of the system and detailed knowledge of its behavior.

A more robust approach for such systems uses a collection of autonomous agents, that each deal with a limited part of the overall control problem. Individual agents can be associated with each sensor or actuator in the material, or with various aggregations of these devices, to provide a mapping between agents and physical location. This leads to a community of computational agents which, in their interactions, strategies, and competition for resources, resemble natural ecosystems [12]. Distributed controls allow the system as a whole to adapt to changes in the environment or disturbances to individual components [9].

Multiagent systems have been extensively studied in the context of distributed problem solving [4, 7, 14]. They have also been applied to problems involved in acting in the physical world, such as distributed traffic control [16], flexible manufacturing [21], the design of robotic systems [17, 24], and self-assembly of structures [19]. However, the use of multiagent systems for controlling smart matter is a challenging new application due to the very tight coupling between the computational agents and their embedding in physical space. Specifically, in addition to computational interactions between agents from the exchange of information, there are mechanical interactions whose strength decreases with the physical distance between them.

In this paper we present a novel control strategy for unstable dynamical systems based on market mechanisms. This is a particularly challenging problem, for in the absence of controls, the physics of an unstable system will drive it rapidly away from the desired configuration. This is the case, for example, for a structural beam whose load is large enough to cause it to buckle and break. In such cases, weak control forces, if applied properly, can counter departures from the unstable configuration while they are still small. Successful control leads to a virtual strengthening and stiffening of the material. Intentionally removing this control also allows for very rapid changes of the system into other desired configurations. Thus an effective way of controlling unstable systems opens up novel possibilities for making structures extremely adaptive.

2. Using Markets for Distributed Control

We assume that sensors and actuators are embedded in this system at various locations. Associated with these devices are computational agents that use the sensor information to determine appropriate actuator forces. The overall system dynamics can be viewed as a combination of the behavior at the location of these agents and the behavior of the material between the agent locations. For the latter, the dynamics consists of high frequency oscillations that are not important for the overall stability [10]. This is because stability is primarily determined by the behavior of the lowest frequency modes. We assume that there are enough agents so that their typical spacing is much smaller than the wavelengths associated with these lowest modes. Hence, to focus on the lower frequency dynamics it is sufficient to characterize the system by the displacements at the locations of the agents only. In this case, the high-frequency dynamics of the physical substrate between agents does not significantly affect overall stability. Instead, the substrate serves only to couple the agents' displacements.

Given the similarities between social organizations and computational ecosystems, it is tempting to test the efficacy and desirability of market mechanisms in distributed control systems. Since market devices such as prices and auctions greatly facilitate resource management in human societies, one would expect them to be similarly useful in managing resources for other control applications. Markets do possess the nice property that consumers with a high demand will always get a high amount of good relatively to the others. This property of markets represents a big advantage in comparison to commonly used control strategies because the distribution of a limited amount of power by a market will automatically consider the importance of the demand without the need of a complicated model.

As in economics, the use of prices provides a flexible mechanism for allocating resources, with relatively low information requirements [8]: a single price summarizes the current demand for each resource.

3. System Description

The system we studied, illustrated in Fig. 1a, consists of n mass points connected to their neighbors by springs. In addition a destabilizing force proportional

to the displacement acts on each mass point. This force models the behavior of unstable fixed points: the force is zero exactly at the fixed point, but acts to amplify any small deviations away from the fixed point. This system can be construed as a linear approximation to the behavior of a variety of dynamical systems near an unstable fixed point, such as the inverted pendulae shown in the Fig. 1b. In the absence of control, any small initial displacement away from the vertical position rapidly leads to all the masses falling over. In this case, the lowest mode consists of all the pendulae falling over in the same direction and is the most rapidly unstable mode of behavior for this system. By contrast, higher modes, operating at shorter length scales, consist of the masses falling in different directions so that springs between them act to reduce the rate of falling.

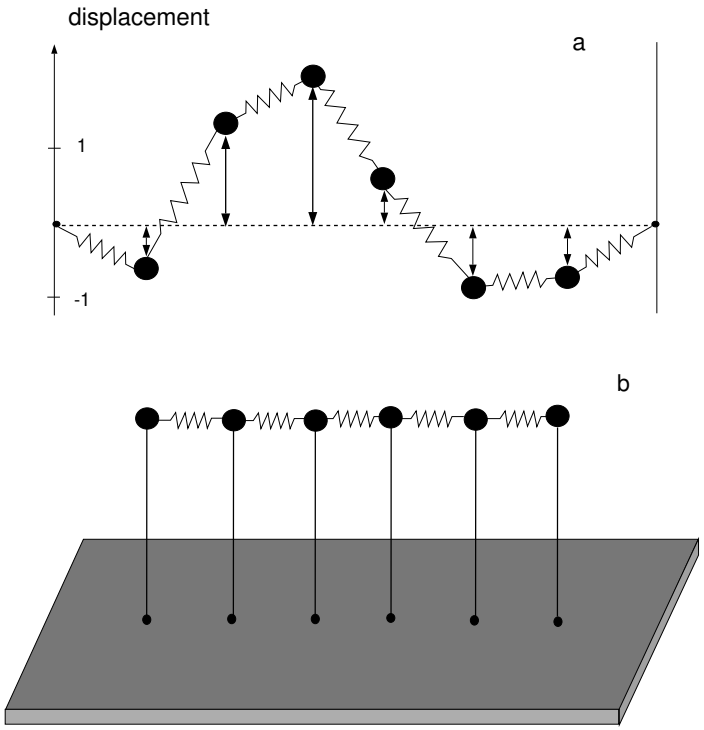

Fig. 1. An unstable dynamical system. a) The unstable chain with the mass points displaced from the unstable fixed point which is indicated by the horizontal dashed line. The masses are coupled to their neighbors with springs, and those at the end of the chain are connected to a rigid wall. b) A chain of upward-pointing pendulae connected by springs as an example of an unstable spatially extended system.

The system's physical behavior is described by

1. the number of mass points n

2. spring constants k
3. a destabilizing force coefficient f
4. a damping force coefficient g

We also suppose the mass of each point is equal to one. The resulting dynamics of the unstable chain is given by[1]:

$$\begin{aligned} \frac{dx_i}{dt} &= v_i \\ \frac{dv_i}{dt} &= k(x_{i-1} - x_i) + k(x_{i+1} - x_i) + f x_i - g v_i + H_i \end{aligned} \tag{1}$$

where x_i is the displacement of mass point i, v_i is the corresponding velocity, and $x_0 = x_{n+1} = 0$ is the boundary condition. The H_i term in Eq. (1) is the additional control force produced by the actuator attached to mass point i. We suppose the magnitude of this control force is proportional to the power P_i used by the actuator, i.e.,

$$|H_i| = \epsilon P_i \tag{2}$$

with the constant ϵ characterizing the actuator's efficiency.

For this system, the control design problem is to determine how hard to push on the various mass points to maintain them at the unstable fixed point. Solving this problem can involve various goals, such as maintaining stability in spite of perturbations typically delivered by the system's environment, using only weak control forces so the actuators are easy and cheap to fabricate, continuing to operate even with sensor noise and actuator failures, and being simple to program, e.g., by not requiring a detailed physical model of the system.

4. A Power Market for Control

Computational markets can successfully coordinate asynchronous operations in the face of imperfect knowledge and changes in the environment [3, 11, 15, 20, 22, 23]. As in economics, the use of prices provides a flexible mechanism for allocating resources, with relatively low information requirements [8]: a single price summarizes the current demand for each resource.

[1] We used a standard ordinary-differential-equation solver for integrating the equations of motion [18].

In principle an omniscient central controller with a perfect system model and unlimited computational capability could optimally control it. However, in practice such detailed knowledge of the system is seldom available. This is especially true in the mass production of smart materials where manufacturing tolerances and occasional defects will cause the physical system to differ somewhat from its nominal specification. Instead, partial information about local changes in the variables is the only reliable source that can be used for controlling smart matter. In particular, price mechanisms perform well compared to other feasible alternatives [5, 13] for a variety of multiagent tasks. It is thus of interest to see how they perform in the new context of multiagent control of smart matter.

In designing a market of computational agents, a key issue is to identify the consumers and producers of the goods to be traded. Various preferences and constraints are introduced through the definition of the agents' utilities. This ability to explicitly program utility functions is an important difference from the situation with human markets. Finally, the market mechanism for matching buyers and sellers must be specified.

In the market control of smart matter treated here, actuators, or the corresponding mass points to which they are attached, are treated as consumers. The external power sources are the producers and as such are separate from consumers. All consumers start with a specified amount of money. All the profit that the producers get from selling power to consumers is equally redistributed to the consumers. This funding policy implies that the total amount of money in the system will stay constant.

In the spirit of the smart matter regime, where control computations are fast compared to the relevant mechanical time scales, we assume a market mechanism that rapidly finds the equilibrium point where overall supply and demand are equal. Possible mechanisms include a centralized auction or decentralized bilateral trades. This equilibrium determines the price and the amount of power traded. Each actuator gets the amount of power that it offers to buy for the equilibrium price and uses this power to push the unstable chain.

Each consumer buys an amount of power, P, that depends on its associated

utility function, which reflects a trade-off between using power to act against a displacement and the loss of wealth involved. While a variety of utility functions are possible, a particularly simple one for agent *i*, expressed in terms of the price of the power, p, and the agent's wealth, w_i, is:

$$U_i = -\frac{1}{2w_i} pP^2 + bP|X_i| \tag{3}$$

where

$$X_i = \sum_{j=1}^{n} a_{ij} x_j \tag{4}$$

is a linear combination of the displacements of all mass points that provides information about the chain's state. The parameter b determines the relative importance to an agent of responding to displacements compared to conserving its wealth for future use.

Actuator i always pushes against the direction that is given by the sign of X_i, i.e., it acts to reduce the value of X_i. In this paper we focus on the simple case of purely local control when $a_{ij} = 1$ when $i = j$ and is 0 otherwise.Thus, consumer i considers only its own displacement x_i. In this case, each actuator always pushes against its own displacement.

We use an ideal competitive market in which each consumer and producer acts as though its individual choice has no affect on the overall price, and agents do not account for the redistribution of profits via the funding policy. Thus a consumer's demand function is obtained by maximizing its utility function as a function of power:

$$\frac{dU_i}{dP} = -p\frac{P}{w_i} + b|X_i| = 0 \;\Rightarrow\; P_i(p) = b|X_i|\frac{w_i}{p} \tag{5}$$

This demand function causes the agent to demand more power when the displacement it tries to control is large. It also reflects the trade-off in maintaining wealth: demand decreases with increasing price and when agents have little wealth. The overall demand function for the system is just the sum of these individual demands, giving

$$P^{\text{demand}}(p) = \frac{b}{p} \sum |X_i| w_i \tag{6}$$

As to the functional form of the supply function, each producer tries to maximize its profit ρ given by the difference between its revenue from selling power and its production cost $C(P)$: $\rho = pP - C(P)$. To provide a constraint on the system to minimize the power use, we select a cost function for which the cost per unit of power, $C(P)/P$ increases with the amount of power. A simple example of such a cost function is

$$C(P) = \frac{1}{2a}P^2 \tag{7}$$

The parameter a reflects the relative importance of conserving power and maintaining stability. We obtain the producer's supply function by maximizing its profit:

$$\frac{d\rho}{dP} = p - \frac{dC}{dP} = 0 \;\Rightarrow\; P(p) = ap \tag{8}$$

For simplicity, we suppose all producers have the same cost, so the overall supply function is then just

$$P^{\text{supply}}(p) = nap \tag{9}$$

From this, we can obtain the price and amount of traded power as the point where the overall demand curve intersects the overall supply curve. Specifically, the price at which power is traded is given by $P^{\text{demand}}(p) = P^{\text{supply}}(p)$. For our choices of the utility and cost functions, this condition can be solved analytically to give

$$p_{trade} = \sqrt{\frac{b}{na}\sum_{i=1}^{n}|X_i|w_i} \tag{10}$$

Given this equilibrium price, agent i then gets an amount of power equal to $P_i(p_{trade})$ according to Eq. (5) and the resulting control force is given by Eq. (2).

We can also consider the case where the amount of power available to the system is limited to $P_{\max}^{\text{global}}$. This hard constraint has the effect of limiting the overall supply function so it becomes

$$P^{\text{supply}}(p) = \begin{cases} nap & \text{if } p < P_{\max}^{\text{global}}/na \\ P_{\max}^{\text{global}} & \text{otherwise} \end{cases} \tag{11}$$

The existence of a power limit makes the supply curve flatten at a maximum value.

The final aspect of the market dynamics is how the wealth changes with time. This is given by

$$\begin{aligned} \frac{dw_i}{dt} &= -pP_i(p) + \frac{1}{n}pP^{\text{demand}}(p) \\ &= -b|X_i|w_i + \frac{b}{n}\sum_{j=1}^{n}|X_j|w_j \end{aligned} \tag{12}$$

because we use the funding policy that all expenditures are returned equally to the agents in the system.

5. Results

We studied a chain composed of 27 mass points, all of them having unit mass and connected by springs with a spring constant of value 1 and damping coefficient 0.1. The destabilizing force coefficient is 0.2, which is sufficient to make the system unstable when there is no control force. All agents start with an initial wealth of 50 money units and we are using the values $a = 5$ and $b = 0.001$ in the cost and utility functions. For definiteness, we choose an initial condition where the single element in the middle of the chain had a unit displacement and all other values were at zero. For the local control, we used $c = 0.2$, which is more than sufficient to ensure stable control when power is unlimited [10]. Power use was related to force in Eq. (2) with $\epsilon = 100$.

With $P_{\max} = 1.2$, Fig. 2 compares the performances of both local and market controls. We show both the total power use $\sum_i P_i$ and the average displacement of the chain $\sum_i |x_i|/n$. As can be seen, for the chosen parameter values the market is able to control the unstable chain in spite of the fact that the power is limited to a global maximum. This limit is reached several times. The local controls (1 and 2), on the other hand, fail in both cases, as seen in the figure. These results were obtained in a simulation run that lasted 20 time units. A longer simulation shows that the overall power usage and average displacements decrease with time for the market control while increasing in the case of the local control strategies.

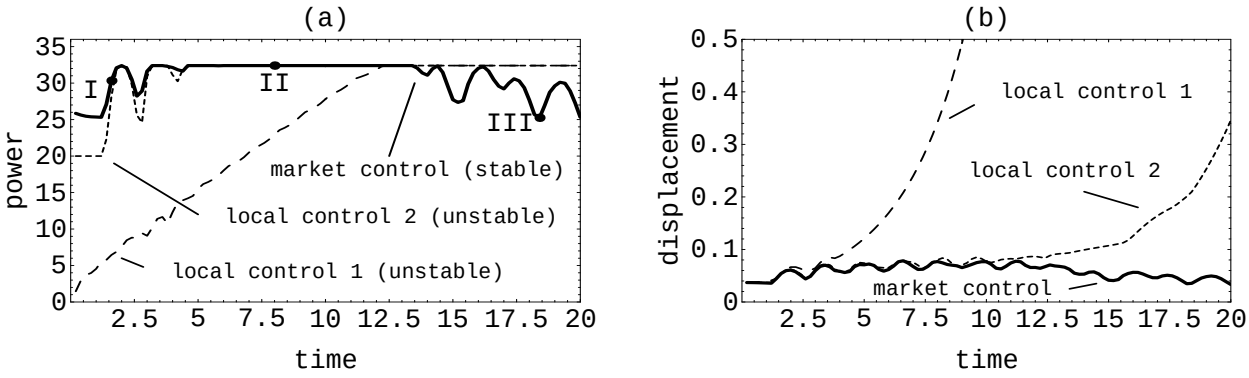

Fig. 2. a) Time development of the overall power usage for a market control (solid) and local controls 1 and 2 (dashed) in the case of limited available power. With the same power limit in all three cases, the market is the only one that can control the unstable chain. Points I, II and III mark the times at which the supply and demand curves intersections are shown in Fig. 3. b) Corresponding time development of the average displacement for a market control and local controls 1 and 2. The market reduces the average displacement with time whereas the local control is not able to prevent it from growing.

Since the power cost function $C(P)$ does not change, the overall supply curve never changes, as shown in Fig. 3 which displays the supply curve and some demand curves for different times. The demand curves depend on the displacements and wealth of the agents. Since these are dynamical variables, overall demand curve changes in time. In addition to the times I, II and III marked in Fig. 2a, we also plot the overall demand curves for later times IV, V and VI. This shows that the amount of traded power decreases with time while the unstable chain is controlled by the market.

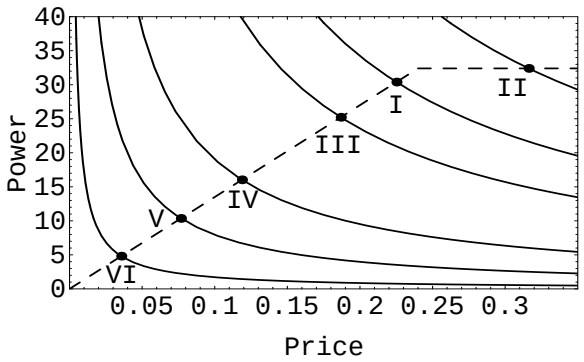

Fig. 3. Overall supply curve (dashed) and the overall demand curves (solid) at times I=1.6, II=8.0, III=18.4, IV=40.0, V=60.0 and VI=90.0 for the market example of Fig. 2..

6. Market Organizations

As we have shown in the previous section, a combined multiagent and market approach offers several advantages compared to commonly used control strategies. We will now show that the underlying organizational structure plays an important role in the performance of the system. Interpreting the a_{ij}'s in equation (3) as a description of how important it is for agent i (actuator) to get information from agent j (sensor) when deciding about how much power to request, we refer to the matrix $\{a_{ij}\}$ as the interaction matrix that represents the organizational structure of the system. In the previous section we used an organizational structure that is similar to the structure of a simple local controller without a market. But any other structure represented by equation (3) is possible. In what follows we will focus on 3 additonal organizational structures that are given by a (1) nearest neighbors, (2) hierarchical and (3) multihierarchical structure [10] and compare their performance in terms of overall used power, average displacement and time to reduce the average displacement below a given value. In Fig.4 we show the interaction matrices for the structures that we studied.

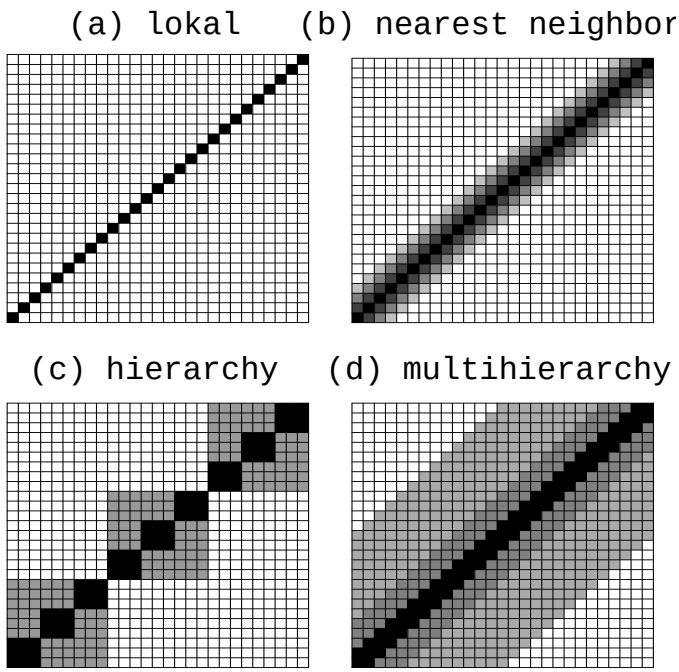

Fig. 4. In a) we see the local structure where every agent listens only to his own sensor and ignores all other agents in the system. The interaction with others in this case is only physically, based on the springs that connect agents. An organization that uses the information of nearest neighbors is shown in b) for a case where the interaction strength decreases linearly with the distance. In Fig c) and d) we show a hierarchical and a multihierarchical organization with 3 levels and a branching ratio of 3.

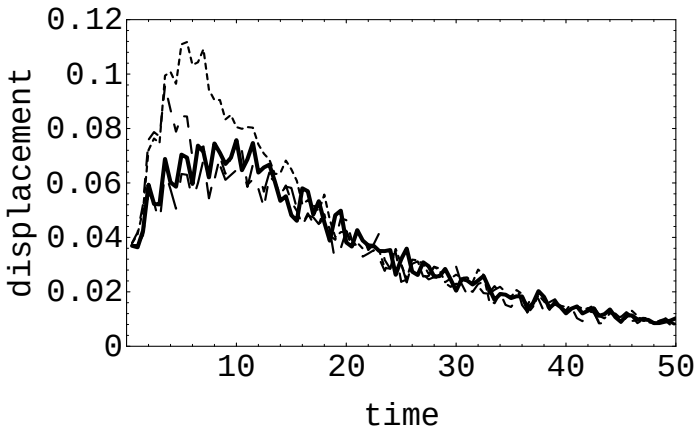

Fig. 5. The Figure shows how different organizational structures succeed in reducing an intial displacement with time. We see that the average displacement differs significantly in the first 15 time units but it does not depend on the organizational structure for later times. The solid curve represents the local structure. The nearest neighbors organization has almost the same shape. But the hierarchy and the multihierarchy show a high displacement in the beginning whereas the multihierarchy performs slightly better.

In Figure 5 we see the time evolution of the average displacement of the different organizational structures that we studied. In the first 15 time units the difference in the average displacement is quite remarkable but for larger times all organizations control the system in more or less the same way and the decrease of the average displacement with time does not any longer depend on the used organizational structure. Nevertheless, the big difference in the first few time units of the controlling efforts gives rise to a potential failure of controlling in case the power is limited.

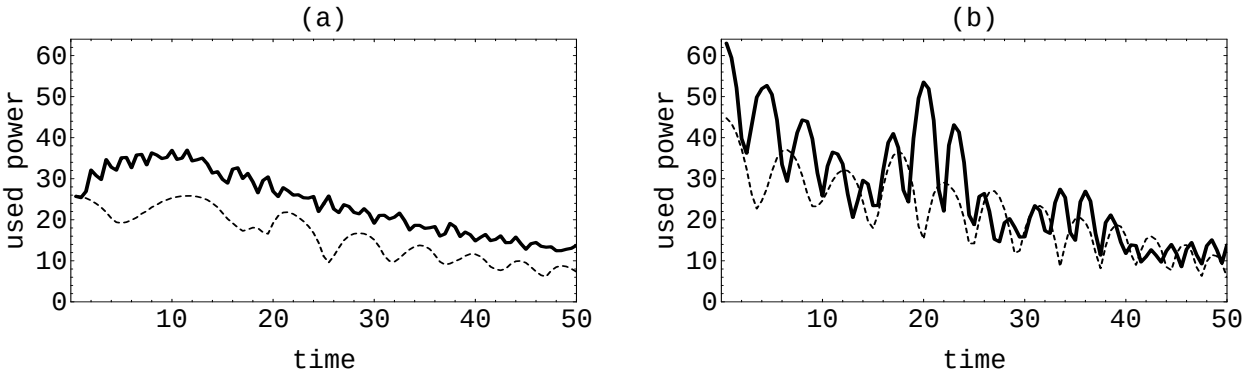

Fig. 6. The Figure shows the time evolution of the power usage of the different organizational structures that we studied. In a) we compare the power usage of a local (solid) and a nearest neighbor (dotted) structure whereas the nearest neighbor structure needs constantly less power. In b) we compare the hierarchical (solid) with the multihierarchical (dotted) structure.

In Figure 6 we compare the overall power usage of the different organizational structures. We observe that the nearest neighbor structure uses the least amount of power to control the unstable system but on the other hand performs very well in reducing the average displacement as can be seen in Figure 5. We explain the observation of high amplitude oscillations in the power usage for the nearest neighbors, hierarchical and multihierarchical structures by the fact that the communication between agents in these organizations results in a more coordinated action of neighbored agents. Thus, it is more difficult for the system to change its current course of action because not only one but several agents act and push in a similar way.

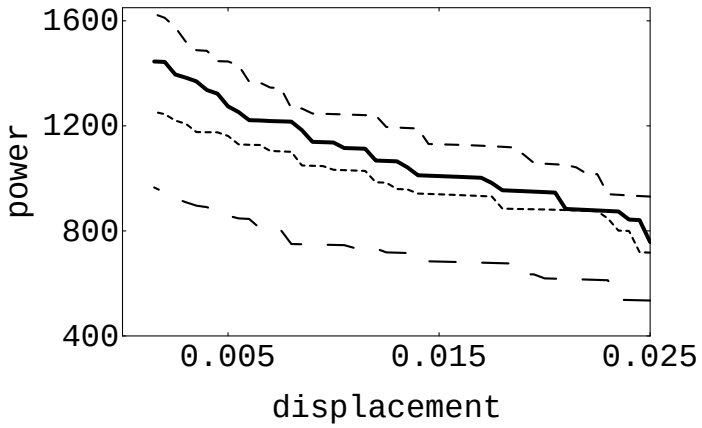

Fig. 7. The Figure shows the overall power usage for the the different organizational structures to reduce the average displacement below a given value. We observe again that the nearest neighbor structure (lower curve) performs better than all the other structures that we studied. The multihierarchy (upper curve) uses the most power whereas the hierarchical structure uses a little less than the local organization (solid curve).

In Figure 7 we compare the different structures by looking at the power usage and the average displacement at the same time. The x-axis denotes the threshold value for the average displacment that has to be met. The y-axis shows the overall power that is used to reduce the average displacement below the given threshold value.

7. Discussion

In this paper we presented a novel mechanism of controlling unstable dynamical systems by means of a multiagent system approach combined with a market

mechanism. We described how we defined consumers and producers utility functions that lead to the overall supply and demand curves and evaluated the price and amount of traded power within the system.

We showed that the market approach is able to control an unstable dynamical system in the case of limited power whereas a traditional local control strategy fails under the same assumptions. We also demonstrated that a market control adapts better to cases when an actuator breaks during the controlling process. These results show that a market control can be more robust than a local control when operating with given power constraints.

We also studied the effectiveness of different market organizations in achieving control, and we noticed that the nearest neighbor structure uses the least amount of power to control the unstable system while also capable of reducing the average displacement of the chain. We can explain this observation by the fact that the physical connection within the system depends on the distance between two interacting agents. A extension of this research consists in the use of a learning mechanism which allows for improving the performance of the system by changing the market organization in time. We are currently studying this.

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